Two-Timing Music with Math:
An Affair New to Controversy?

Soham Munim

Images of the mathematics in handwriting are the author's own handwritten notes.

Images of notebooks quoting The Mother & Sri Aurobindo are from

the author's own notebook back from school, provided to him at SAICE.

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PREFACE

Here is an account of my handwritten notes that I had written over a decade ago, regarding a mathematical approach using the fundamentals in Permutation & Combination, and applying them to generate various scales using the musical notes; most importantly formulating the groupings of those scales defined by the sizes of the gaps between the musical notes i.e. **t** & **s** (Tone & Semi-tone) or simply in terms of s (as t = 2s), as well as the Order & Rotation of those gaps i.e. a scale whose total size is 6t or 12s could be further broken down to 12s = 4s + 3s + 2s + s + 2s which in terms of musical notes, the same would translate to S G P D n **S** or Sa, Ga, Pa, D, Komal Ni, upper Sa, just as 1 order among others by shuffling i.e. 3s + 4s + 2s + s + 2s which gives S g P D n **S.** This is not the same as the same Order in Rotation i.e. by putting 4s at the end of the sequence and starting with 3s, thereby giving us the scale S g m M d **S** with the concept of this rotation similar to the modes of a scale i.e. Ionian, Dorian, Phrygian, Lydian, etc in Western Classical Music, although the rudimentary theory of modes does not cater to application of notes with a 3s & 4s gaps i.e Rag Bhairav which goes **S(4s)G(s)m(2s)P, G(s)m(3s)d(3s)N(s)S**; or coming back with **S(-s)N(-3s)d(-s)P(-2s)m(-s)G(-s)m(-4s)r(-s)S.**

The Permutation & Combination in the order of notes exists in a slightly different manner in Carnatic Music owing to "full scales" or 7 notes in the scale as the main combinations or parent scales in their Melākartā of 72 (36 with m + 36 with M and 6x6 combinations with rRgGdDnN). I had learnt Sitar in the Hindustāni Format where Thāts were given the importance as parent scales but not encompassing several Ragas as they are not mathematically in place. As I had learnt Rāgas further with Pandit Shyamal Chattopadhyay I had realised that it is next to impossible for mathematical classification of Rāgas leading to a lot of ambiguity as far as Rāgadāri or the grammar (refer to Bhairav above) is concerned with a particular order of notes going up the scale and another order in returning and therefore I will never try to classify highly defined Rāgas in a scale system of parent scales beyond my accepting the ultimate definition of those Rāgas as a collectives of particular phrases such as the Pakad, the Chalan & the Nyāsa.

But my mathematical efforts in dabbling wth scales and observing sequences of notes also led me to formulate the following research in trying to get to a system of scales that is mathematically sound and interestingly generates so many scales I may have never come across sticking to the existing scale systems. In this newer system that I propose, we can see that how many of the familiar scales from the older system can be clubbed together with completely unfamiliar scales. For example:

Let's consider the Kāfi Thāt S R g m P D n S from Hindustani or the Dorian Mode from the Modal system of Western Classical Music as denoted in gaps such as 2s+s+2s+2s+2s+s+2s and see the rotations that can be generated using this sequence:

1. **2s+s+2s+2s+2s+s+2s** S R g m P D n S Kāfi Thāt
2. **s+2s+2s+2s+s+2s+2s** S r g m P d n S Bhairavi Thāt
3. **2s+2s+2s+s+2s+2s+s** S R G M P D N S Kalyān Thāt
4. **2s+2s+s+2s+2s+s+2s** S R G m P D n S Khamāj Thāt
5. **2s+s+2s+2s+s+2s+2s** S R g m P d n S Asāvari Thāt
6. **s+2s+2s+s+2s+2s+2s** S r g m M d n S   **?**
7. **2s+2s+s+2s+2s+2s+s** S R G m P D N S Bilāwal Thāt or the natural scale(!)

In other words, the entire 7 modes in the Western Music Theory and 6 Thāts of Hindustani, can all fall under a single Order with all its Rotations! Therefore what are the possibilities in store with all the Rotations of different Orders and more importantly how do we count them?

I was thrilled to find the mathematical formula **working it out myself without being influenced by any academic paper** as I did not go further into academics as I had to pursue Sitar after beginning to learn it at age 17! I had considered several times to write a book in music, but did not pursue that ambition as the formula that I had developed was simple and beautiful – any overuse of music theory to present it might just add to more "academic drama" while my profession has been that of a hardcore live performer learning so much from life! To be fair, I cannot ascertain that nobody else has come up with this, as the math is elementary although very interesting in its application, and especially when I did not bother to publish this for over a decade as I was too lazy to learn coding to write a program / software (to display the scales) given that the amount of syntax one has to remember while coding is anti-thesis to my life as a performer on Sitar who remembers the bare minimum onstage deriving the math in rhythm on the spot as I improvise, and at times even creating compositions on the fly! Not having the ambition to write a book wasn't the only reason for the delay in publishing this.

In case this idea is already published, I will still have the joy of having discovered it all by myself in applying the **fundamentals of Permutation & Combination** that I had learnt under Mr. Bharg Shah as my teacher in Math, **just before he had kicked me out of his class!** Not to dissect what had happened 20 years ago, but **on a lighter note,** I thought let me remind him of how **I needed to use only this little bit of elementary math** in combining with my knowledge in music **to produce something unique** and worth remembering, purely **for the love of math & music together; even when I had to learn** that elementary math **from someone who wished me no future in math!** And thus we can all broadly reflect upon all those who cherish their certificates other than for their getting any jobs, with some, as redundant as garnering it for their social impression in helping them in finding their prospective brides/ grooms! **Statistically speaking,** out of 12 of my batchmates in Math (6+6 in 2 classes), we have only 1 PhD in Math, 1 PhD in Physics and 1 working seriously as a programmer in the IT Sector (not those fake jobs calling them IT!). **Did the rest 9 even use math in their life especially CERTIFIED as "deserving students"?! Thanks to whom?**

I thank sincerely, Ms. Glory Pal for providing me with some **old dairy reports** that were not required by them as **rough paper for me to use,** further validating my research as being "down to earth" (with the presence of cows!), during a visit to Gloria Land (Sri Aurobindo Ashram) where I received my Eureka moment of inspiration(!), **not needing Bharg bhai nor his notebook for reference unlike his safeguards(!)** as I worked it out **independently** late into the night to arrive at this solution! Naturally Glory was the 1st person with whom I could share this work during her early "Good Morning" & before my late "Good Night" as I proceeded to sleep shamelessly through the day on an early rising farm! **I also thank** my alumni from SAICE, Dr. Aurobrata Ghosh (ex-IIT Delhi + places that I don't know) & my former SAICEan classmate, Dr. Namrata Patel (ex-Sorbonne University) for lending me their ears in order to cross-check with my math. **And last but not the least I thank both,** my Guru Pt. Basab Sen for teaching me Sitar and infusing the elements of Rāgadāri, as well as Sangeetāchārya Pt. Shyamal Chattopadhyay for guiding me on Rāgadāri "beyond this earth and well into space"; all of which has contributed to my understanding of how Rāgas cannot be classified mathematically in a system of scales, while I'm free to formulate my own system of scales purely in the interest of music and math together.

<div align="right">

*#sitarwonderboy & Creator of the SiGui* **Soham Munim**
A-Grade in Sitar in Akashwani (AIR)
& Doordarshan (TV); ICCR Empanelled.

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6th January 2023, Puducherry.

"If in any Rāga, you do not get the reflection of another Rāga, be it by shifting the root note (Sa) to any other note in that Rāga; then that Rāga will not sound harmonious to our ears."
~ Sangeetacharya Pt. Shyamal Chattopadhyay

Sri Aurobindo
International Centre of Education
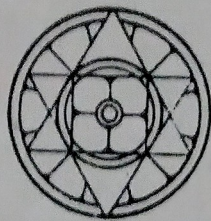
The children should be helped to grow up into straightforward, frank, upright and honourable human beings ready to develop into divine nature.

Sri Aurobindo

Name..

Subject................................ Section ....

Date ....

**SRI AUROBINDO ASHRAM
PONDICHERRY**

PRIÈRE DES ÉLÈVES

Fais de nous les guerriers héroïques que nous aspirons à devenir, pour livrer avec succès la grande bataille de l'avenir qui doit naître, contre le passé qui veut durer; afin que les choses nouvelles puissent se manifester et que nous soyons prêts à les recevoir.

La Mère

STUDENTS' PRAYER

Make of us the hero warriors we aspire to become. May we fight successfully the great battle of the future that is to be born, against the past that seeks to endure; so that the new things may manifest and we be ready to receive them.
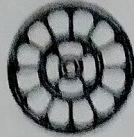
The Mother

DECLARATION
(À répéter chaque jour par tous les élèves)

Ce n'est pas pour notre famille, ce n'est pas pour avoir une situation, ce n'est pas pour gagner de l'argent, ce n'est pas pour obtenir un diplôme, que nous étudions.

Nous étudions pour apprendre, pour savoir, pour comprendre le monde, et pour la joie que cela nous donne

Apprendre pour savoir, etudier pour connaître les secrets de la nature et de la vie, s'éduquer pour faire croître sa conscience, se discipliner pour devenir maître de soi, pour surmonter ses faiblesses, ses incapacités et ses ignorances, se preparer a avancer dans la vie vers un but plus noble, plus vaste, plus genereux et plus vrai...

Nous voulons, ici, seulement ceux qui aspirent à une vie plus haute et meilleure, ceux qui ont soif de connaissance et de perfection, ceux qui regardent ardemment vers un avenir plus totalement vrai.

<div align="right">La Mere</div>
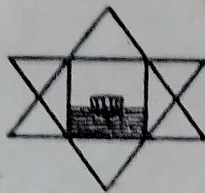
The Yogin's aim in the practical sciences, whether mental and physical or occult and psychic, should be to enter into the ways of the Divine and his processes, to know the materials and means for the work given to us so that we may use that knowledge for a conscious and faultless expression of the spirit's mastery, joy and self-fulfilment. The Yogin's aim in the Arts should not be a mere aesthetic, mental or vital gratification, but, seeing the Divine everywhere, worshipping it with a revelation of the meaning of its works, to express that One Divine in gods and men and creatures and objects.

<div align="right">Sri Aurobindo</div>

S 25 25 25 S 25 25

(01) S S S - - - -
    S S - S - - - R(04)
    S S - - S - - (R(03)
    S S - - - S - R(02)
    S S - - - - S (R(01)

(02) S - S S - - -
    S - S - S - - O5
    S - S - - S - R(05)
    S - S - - - S R(04)

(03) S - - S S - -
    S - - S - S - R(05)
    S - - S - - S R(03)
    S ~~___~~ S

R(01)
(04) S - - - S S -
    S - - - S - S R(02)

    S - - - - S S (05)

- S S S - - - (R(01)
- S S - S - - R(04)
- S S - - S - (R(03)
- S S - - - S R(02)

= S - S S - - R(02)
- S - S S S - R(05)
- S - S - - S (R(05)

- S - - S S - R(03)
- S - - S - S (R05)
- S - - - - S S R(04)

- - S S S - - (R(01)
- - S S - S - R(04)
- - S S - - S R(03)

- - S - S S - R(02)
- - S - S - S R(05)
~~___~~

- - S - - S S R(03)

- - - S S S - (R(01)
- - - S S - S R(04)

- - - S - S S $R(02)
~~___~~

- - - - S S S (R(01)

$$(5+4+3+2+1) + (4+3+2+1) + (3+2+1) + (2+1) + 1$$

1)5 + 2(4) + 3(3) + 4(2) + 5(1)

Getting closer... Too crowded? Don't worry the cows have blessed me!

Herd No.	:	6877
Name	:	TULKA JY50 x INDA 25 x THARI 12.5 x HFI2.5
Date of birth	:	28-08-2003
Dame	:	ULKA, THARI x HF3 x JY
Sire	:	CBJ 3449
Date of first calving:	23-09-05
Date of last calving :	26-08-06	F.C.
Date and nature of service given:	Nov end to be Revised
1. N.S. Steere given at 30.9.06
Date of confirmed pregnancy	:	15th Jan 07
Sire responsible pregnancy	:	Steere Bull
Expected date of calving	:	June 07, Delivered a l. Calf 30th
Placenta at previous calving	:	Normal
Remarks (medication etc)	:

1.
2. $3 \times 2 \times 1$
3.
4.
5.
6.
7. $\dfrac{4!}{3!}$
8.
9. $7 \times 6 \times 5 \times \ldots$
10.

$35 \times 4 = 140$

$5 + 4 + 3 + 2 + 1$
$4 + 3 + 2 + 1$
$3 + 2 + 1$
$2 + 1$

$5 \times 4 = 20$

$C_7^2 \times C_5^1$

$C_7^4 \times C_3^2$

$C_7^3 \times C_4^3 = 140$

$\dfrac{7!}{2! \, 5!} = \dfrac{42}{2} = 21$

$\dfrac{7!}{2! \, 4!} = \dfrac{7 \times 6 \times 5}{2} = 165$

Delivered a l. Calf at 30.6.07

$\dfrac{7!}{3!(4!)}$

$7 \times 6 \times 5 \ldots$

$S+S+3S+3S+2S+2S$

🙏🐮🐄🐮🙏

Yes! Totally down-to-earth research...

$$\frac{n!}{x_1!\,(n-x_1)!} \times \frac{(n-x_1)!}{x_2!\,(n-x_1-x_2)!} \times \frac{\left(n-x_1-x_2\over r_2\right)!}{x_3!\,(n-x_1-x_2-x_3)!} \cdots = n!\Big/ x_1! \times x_2! \times x_3! \times x_4! \,(n-x_1-x_2-x_3-x_4)!$$

$$\times \frac{(n-x_1-x_2-x_3)!}{x_4!\,(n-x_1-x_2-x_3-x_4)!}$$

Herd No. : G910

Name : yohanna GIR 40 × SW 15 × HF 20 × JY 25

Date of birth : 25.11.2004.

$$5! \Big/ x_1! \times 2! \times x_1! \times x_1! = 5 \times 4 \times 3$$

Dame : G 536. Malavika.

Sire : B 007. Srenit Bull.

Date of first calving : 23.06.2007.

Date of last calving : 23.06.2007.

SS SS SS SS Sh
SSS SS S Sh
SSSS SS SS

Date and nature of service given:

1. On Heat dt. 16.08.07 A.I. Mirinda.
2. N.S. Sitara dt. 29.10.07
3.
4. $C_5^5 \times C_4^4 \times C_3^3 \times C_2^2 \times C_1!$
5. $x_1, x_2, x_3 \cdots x^{-1}$

6. $\cos 2\varphi$

$$15 = \frac{30}{2} = \frac{7\times 2!}{6\times 5\times 4!} = \frac{6!}{2!} = \frac{7!}{9}$$

7.
8.
9. From a finite set once, once only once.
10. permutation is a seqn. of each element.

$$n(n-1)\,(n-2)\times\cdots\times 4\times 2\times 1 = n!$$

perm of n disct objects is

SSS SSS Sh
SSS SS Sh S
Sh SSSSSh SSS SS SS
SS Sh Sh S SS
S Sh S Sh
S S S S S

Date of confirmed pregnancy :

Sire responsible pregnancy :

Expected date of calving :

Placenta at previous calving : Normal (First calving)

Remarks (medication etc) :

Check dt. 23rd July 2007.
Problem - Both ovaries small n smooth.
Medicine - Iodium - 30/ 15 days / once a day.
Sepia - 200/ 3 weeks / once a week.

$(f \cdot g)' = f'g + fg'$

$(f/g)' = \dfrac{f'g - fg'}{g^2}$

$y = ax + b \qquad y = f(x) \cdot g(x)$
$\Delta y = a\Delta x + b \qquad \Delta y = f(x)\cdot g(x) = f(x-\Delta x)\cdot g(x-\Delta x)$
$y - \Delta y = a(x-\Delta x) \Rightarrow \Delta y = y - a(x-\Delta x)$
$= a(x - \Delta x + \Delta x) \Rightarrow \lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x} = a$

Variables — $S, 2S, 3S, 4S$ Scale $= S \, 12S$

min $5H$ max $7\frac{1}{2}$ ~~min 5th max 7th~~

Group > orders > rotation

<u>7 notes</u>

G_1 — ~~5S~~ $2 \times S$ & $5 \times 2s$ → 3 orders each ~~of~~ $\overset{order}{}$ rotations $C_7^2 = 21$
$21/7 = 3$ orders

G_2 — $3 \times S$ & $3 \times 2s$ & $1 \times 3s$ → $C_7^3 \times C_4^3 = 140$, $140/7 = 20$ orders

G_3 — $4 \times S$ & $2 \times 3s$ & $1 \times 2s$ → $C_7^4 \times C_3^2 = 105$, $105/7 = 15$ orders

G_4 — $4 \times S$ & $2 \times 2s$ & $1 \times 4s$ → $C_7^4 \times C_3^2 = 105$, $105/7 = 15$ orders

<u>6 notes</u>

G_1 — $4 \times S$ & $2 \times 4s$ → $C_6^4 = \frac{6!}{4! \, 2!} = \frac{6 \times 5}{2} = \frac{30}{2} = 15$, $15/6 = 5/2$ orders ($O_5 = 3R$)

G_2 — $3 \times S$ & $3 \times 3s$ → $C_6^3 = \frac{6!}{3! \, 3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$, $20/6 = 10/3$ orders ($O_4 = 2R$

G_3 — $3 \times S$ & $1 \times 2s$ & $1 \times 3s$ & $1 \times 4s$ → $C_6^3 \times C_3^1 \times C_2^1 = \frac{6!}{3! \, 3!} \times \frac{3!}{1! \, 2!} \times \frac{2!}{1! \, 2!} = \frac{6 \times 5 \times 4}{2 \times 2} = 30$

G_4 — $2 \times S$ & $3 \times 2s$ & $1 \times 4s$ → $C_6^2 \times C_4^3 = \frac{6!}{2! \, 4!} \times \frac{4!}{3! \, 1!} = \frac{6!}{2! \, 3!} = \frac{6 \times 5 \times 4 \times 3!}{2 \times 3!} = 60$

G_5 — $2 \times S$ & $2 \times 2s$ & $2 \times 3s$ → $C_6^2 \times C_4^2 = \frac{6!}{2! \, 4!} \times \frac{4!}{2! \, 2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2! \times 2!} = 90$

G_6 — ~~S~~ $1 \times S$ & $4 \times 2s$ & $1 \times 3s$ → $C_6^1 \times C_5^4 = \frac{6!}{1! \, 5!} \times \frac{5!}{4! \, 1!} = \frac{6 \times 5 \times 4!}{4!} = 30$

~~G_8~~ — ~~1 RS S~~ ~~$6 \times 2s$~~ → $C_6^6 = \frac{6!}{6! \, 0!} = 1$, ~~$1/6$~~ → 1 scale $= 1$or & $1R$ (Diminished)

<u>5 notes</u>

G_1 — $2 \times S$ & $1 \times 2s$ & $2 \times 4s$ → $C_5^2 \times C_3^1 = 30 \Rightarrow \frac{30}{5} = 6$ or

G_2 — $2 \times S$ & $2 \times 3s$ & $1 \times 4s$ → $C_5^2 \times C_3^2 = 30 \Rightarrow \frac{30}{5} = 6$ or

G_3 — $1 \times S$ & $1 \times 2s$ & $3 \times 3s$ → $C_5^1 \times C_4^1 = 20 \Rightarrow \frac{20}{5} = 4$ or

G_4 — $1 \times S$ & $2 \times 2s$ & $1 \times 3s$ & $1 \times 4s$ → $C_5^1 \times C_4^2 \times C_2^1$
$= 5 \times 6 \times 2 = 60$
$\Rightarrow 60/5 = 12$ or

G_5 — $4 \times 2s$ & $1 \times 4s$ → $C_5^4 = 5 \Rightarrow \frac{5}{5} = 1$ or

G_6 — $3 \times 2s$ & $2 \times 3s$
→ $C_5^3 = \frac{5 \times 4}{2} = 10 \Rightarrow \frac{10}{5} = 2$ or

①

Scale = 12s ; Group Variables = s, 2s, 3s, 4s.

Num of notes in a scale : min 5 & max 7 w/ gaps

A group consists of variables such that the sum of all variables in the group is equal to the value of the scale (12s), and the number of gaps in a scale, $5 \geq n \geq 7$, are all integer values.

There are groups in scales with 5, 6 & 7 gaps, can be denoted as nG_i, where n is the number of gaps in the scale and i, the indices of a group. (Index)

Any combination of variables that is distinct and is equal to 12s, forms a group that's distinct, ie, $7G_1 = \{2 \times s + 5 \times 2s\}$ via combination of ; $7G_2 = 3 \times s + 3 \times 2s$ e + 1 x 3s, etc.

Any distinct combination of variables can have several distinct permutations in the order of variables arranged, forming distinct sequences. These distinct sequences can be denoted as $O_j(nG_i)$, where O is the distinct order of the Group and j the indices of the order, ie, $O_1(7G_1) = $ s s 2s 2s 2s 2s 2s ; (Index)

$$O_2(7G_1) = s\ 2s\ s\ 2s\ \ 2s\ 2s\ 2s.$$

Each order in a group can have rotations similar to the concept of modes of a scale in Western Classical music.* The number of rotations in each order is equal to the number of gaps in the order.

$$R_1[O_1(7G_1)] = s\ s\ 2s\ 2s\ 2s\ 2s\ 2s$$

$$R_2[O_1(7G_1)] = s\ 2s\ 2s\ 2s\ 2s\ 2s\ s,\ etc.$$

* The rotation R of an order O_j of a group G_i can be denoted as $R^k[O_j(G_i)]$

|| Therefore a scale can be referred to as $R_k[O_j(nG_i)]$.

Counting of Orders and Rotations in a Group (2)

All the rotations in a group are the total number of permutations of the distinct combination of variables that form a group. So, if the group, nG_i, has p^k rotations, then the total number of orders in the group is

$$T = P/n$$

Consider the group $7G_2 : (3 \times S) + (3 \times 2S) + (1 \times 3S)$. So $p = C_7^3 \times C_4^3 = \dfrac{7!}{3! \, 4!} \times \dfrac{4!}{3! \, 1!}$

$= \dfrac{7 \times 6 \times 5 \times 4 \times 3!}{6 \times 3!} = 140 \Rightarrow T = P/n = \dfrac{140}{7} = 20$.

Generalization of the procedure in counting of orders and rotations in a group:

$nG_i : (x_1 \times S) + (x_2 \times 2S) + (x_3 \times 3S) + (x_4 \times 4S) \Rightarrow p = C_n^{x_1} \times C_{n-x_1}^{x_2} \times C_{n-(x_1+x_2)}^{x_3} \times \dots$ or

$P = C_n^{x_1} \times C_{n-x_1}^{x_2} \times C_{n-(x_1+x_2)}^{x_4}$ or $C_n^{x_1} \times C_{n-x_1}^{x_3} \times C_{n-(x_1+x_2)}^{x_4}$) or all the permutations those as well!

So if $p = C_n^{x_1} \times C_{n-x_1}^{x_2} \times C_{n-x_1-x_2}^{x_3} \times C_{n-x_1-x_2-x_3}^{x_4}$

$= \dfrac{n!}{x_1!(n-x_1)!} \times \dfrac{(n-x_1)!}{x_2!(n-x_1-x_2)!} \times \dfrac{(n-x_1-x_2)!}{x_3!(n-x_1-x_2-x_3)!} \times \dfrac{(n-x_1-x_2-x_3)!}{x_4!(n-x_1-x_2-x_3-x_4)!}$

$= \dfrac{n!}{x_1! \, x_2! \, x_3! \, x_4! \, 0!}$ as $x_1 + x_2 + x_3 + x_4 = n$

And finally $T = P/n = \dfrac{1}{n} \times \dfrac{n!}{x_1! \, x_2! \, x_3! \, x_4!} = \dfrac{1}{n} \times \dfrac{n(n-1)!}{x_1! \, x_2! \, x_3! \, x_4!} = \dfrac{(n-1)!}{x_1! \, x_2! \, x_3! \, x_4!}$

At this point, we have the tool to generate any groups of scales by replacing the variables. To verify this tool to display all the scales it's to code a program. Since I was too lazy to learn coding, I'm going to show a few examples of the this tool by writing out some Orders of Groups.

Examples of some 6 notes scale Groups of Orders with their Rotations...

6 notes

O₁(G₁) –
↑S S S S 4s 4s → S R R G G D Ś
S S S 4s 4s S → S R R G P N Ś
S S 4s 4s S S → S R R M N N Ś
S 4s 4s S S S → S R M D N N Ś
4s 4s S S S S → S G D D N N Ś
4s S S S S 4s → S G M M P D Ś

O₂(G₁) –
S S S 4s S 4s → S R R G P D Ś
S S 4s S 4s S → S R R M P N Ś
S 4s S 4s S S → S R M M N N Ś
4s S 4s S S S → S G M D N N Ś
S 4s S S 4s 4s → S R M M P D Ś
4s S S S 4s S → S G M M P N Ś

O₃(G₁) –
S S 4s S S 4s → S R R M P D Ś
S 4s S S 4s S → S R M M P N Ś
4s S S 4s S S → S G M M N N Ś

6 notes

G₁ – 4 x S & 2 x 4s → $C_6^4 = \frac{6!}{4!\,2!} = \frac{6\times5}{2} = \frac{30}{2} = 15$. 15/6 = 5/2 orders (O₃=3R)

G₂ – 3 x S & 3 x 3s → $C_6^3 = \frac{6!}{3!\,3!} = \frac{6\times5\times4}{3\times2\times1} = 20$. 20/6 = 10/3 orders (O₄ = 2R)

G₃ – 3 x S & 1x2s & 1x3s & 1x4s → $C_6^3 \times C_3^1 \times C_2^1 = \frac{6!}{3!\,3!} \times \frac{3!}{1!\,2!} \times \frac{2!}{1!\,1!} = \frac{6\times5\times4}{2\times2} = 30$

G₄ – 2 x S & 3 x 2s & 1 x 4s → $C_6^2 \times C_4^3 = \frac{6!}{2!\,4!} \times \frac{4!}{3!\,1!} = \frac{6\times5\times4\times2!}{2\times2} = 60$

G₅ – 2 x S & 2 x 2s & 2 x 3s → $C_6^2 \times C_4^2 = \frac{6!}{2!\,4!} \times \frac{4!}{2!\,2!} = \frac{6\times5\times4\times3}{2\times3!} = 90$

G₆ – 1 x S & 4 x 2s & 1 x 3s → $C_6^1 \times C_5^4 = \frac{6!}{1!\,5!} \times \frac{5!}{4!\,1!} = \frac{6\times5\times4!}{4!} = 30$

G₇ – +3s & +6 x 2s → $C_6^6 = \frac{6!}{6!\,0!} = 1$. 1 scale = 1or & 1R (Diminished)

5 notes

At first I was worried with 3 rotations only in the 3rd Order on the LHS. But a look back at my problem solving actually dispelled my concern

O₁(G₂) –
S S S 3s 3s 3s → S R R G M D Ś
S S 3s 3s 3s S → S R R M D N Ś
S 3s 3s 3s S S → S R G P N N Ś
3s 3s 3s S S S → S G M D N N Ś
3s 3s S S S 3s → S G M P D D Ś
3s S S S 3s 3s → S G G M M D Ś

O₂(G₂) –
S S 3s S 3s 3s → S R R M M D Ś
S 3s S 3s 3s S → S R G M D N Ś
3s S 3s 3s S S → S G G P N N Ś
S 3s 3s S S 3s → S R G P D D Ś
3s 3s S S 3s S → S G M P D N Ś
3s S S 3s S 3s → S G G M D D Ś

O₃(G₂) –
S 3s S S 3s 3s → S R G M M D Ś
3s S S 3s 3s S → S G G M D N Ś
S S 3s 3s S 3s → S R R M D D Ś
S 3s 3s S 3s S → S R G P D N Ś
3s 3s S 3s S S → S G M P N N Ś
3s S 3s S S 3s → S G G P D D Ś

O₄(G₂)
S 3s S 3s S 3s – S R G M D D Ś
3s S 3s S 3s S – S G G P D N Ś

as 5/2 orders = 2 Full Orders with 6R each + half Order in a 6 notes scale gives you exactly 3R. See O3(G1).

Even more impressive was O4(G2).. 10/3 Orders in G2 means 3 Full Orders with 6R + 1/3 Order in a 6 notes scale gives you exactly 2R.

Truely a great joy to know that the tool I had developed earlier (in 1&2) pre-verification was correct!!

Examples of some 7 notes scale Groups of Orders with their Rotations...
Sorry about some errors in my handwritten notes not in my equation/ algebra
earlier, but here in listing out the scales which is *really* a software's job!
So it's 7G1 in O2 & O3, not 7G2

$$\frac{7!}{2!(5)} \quad \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 5 \times 4 \times 3 \times 2} \qquad \text{7notes} \quad 7G_1$$

$O_1(7G_1)$
S S 2s 2s 2s 2s 2s → S R R G M̈ D̲ N̲ S
S 2s 2s 2s 2s 2s S → S R̲ G M P D N S̈
2s 2s 2s 2s 2s S S → S R G M̈ D̲ N N S
2s 2s 2s 2s S S 2s → S R G M̈ D D N̲ S
2s 2s 2s S S 2s 2s → S R G M̈ P D N̲ S̈
2s 2s S S 2s 2s 2s → S R G M M̈ D̲ N S̈
2s S S 2s 2s 2s 2s → S R G̲ G M̈ D̲ N S̈

___

● S 2s S 2s 2s 2s 2s → S R̲ G G M̈ D̲ N S
$O_2(7G_1)$
2s S 2s 2s 2s 2s S → S R G M P D N S
S 2s 2s 2s 2s S 2s → S R̲ G M P D N̲ S̈
2s 2s 2s 2s S 2s S → S R G M̈ D D N S̈
2s 2s 2s S 2s S 2s → S R G M̈ P D N̲ S̈
2s 2s S 2s S 2s 2s → S R G M P D̲ N S̈
2s S 2s S 2s 2s 2s → S R̲ G M M̈ D̲ N̲ S̈

___

$O_3(7G_1)$
S 2s 2s S 2s 2s 2s → S R̲ G M M̈ D̲ N S̈
● 2s 2s S 2s 2s 2s S → S R G M P D N S̈
2s S 2s 2s 2s S 2s → S R̲ G̲ M P D N̲ S̈
S 2s 2s 2s S 2s 2s → S R̲ G M P D̲ N̲ S̈
2s 2s 2s S 2s 2s S → S R G̲ M̈ P D N S̈
2s 2s S 2s 2s S 2s → S R G M P D N S̈
2s S 2s 2s S 2s 2s → S R G̲ M P D̲ N̲ S̈

7 notes

$G_1 - 2 \times S \ \& \ 5 \times 2s \rightarrow 3$ orders each $\overset{order}{\text{at}}$ rotations $c_7^2 = 21$
$21/7 = 3$ orders

$G_2 - 3 \times S \ \& \ 3 \times 2s \ \& \ 1 \times 3s \rightarrow c_7^3 \times c_4^3 = 140. \quad 140/7 = 20$ orders

$G_3 - 4 \times S \ \& \ 2 \times 3s \ \& \ 1 \times 2s \rightarrow c_7^4 \times c_3^2 = 105. \quad 105/7 = 15$ orders

$G_4 - 4 \times S \ \& \ 2 \times 2s \ \& \ 1 \times 4s \rightarrow c_7^4 \times c_3^2 = 105. \quad 105/7 = 15$ orders

Check Reference: 7G1 = 21/7 = 3 Orders

O₅ (Tivra)

~~S 2s 3s 2s 2s 3s S S~~
~~2s S 2s 3s 3s S S~~

S 2s S 2s S 2s 3s
2s S 2s S 2s 3s S
S 2s S 2s 3s S 2s
2s S 2s 3s S 2s S
S 2s 3s S 2s S 2s
2s 3s S 2s S 2s S
3s S 2s S 2s S 2s

—

O₆ (Tivra) S S S 2s 2s 3s 2s
S S 2s 2s 3s 2s S
S 2s 2s 3s 2s S S
2s 2s 3s 2s S S S
2s 3s 2s S S S 2s
3s 2s S S S 2s 2s
2s S S S 2s 2s 3s

—

O₇ (Tivra) S S S 2s 3s 2s 2s
S S 2s 3s 2s 2s S
S 2s 3s 2s 2s S S
2s 3s 2s 2s S S S
3s 2s 2s S S S 2s
2s 2s S S S 2s 3s
2s S S S 2s 3s 2s

—

O₈ (Tivra) S S S 3s 2s 2s 2s
S S 3s 2s 2s 2s S
S 3s 2s 2s 2s S S
3s 2s 2s 2s S S S
2s 2s 2s S S S 3s
2s 2s S S S 3s 2s
2s S S S 3s 2s 2s

S r g G M P D **S**
S R g m M d N **S**
S r g G M D n **S**
S R g m d D N **S**
S r g M P D n **S**
S R m M d D N **S**
S g G M P D n **S**

So you get the idea...
I would've loved to type out all the scales,
however I have to respect my body while
spending time with devices, so I'm not
completing the rest here. But you're free
to think of that too as my laziness!

Let's just say that I'm finally getting at least
the heart if not the brain of a coder in expecting
the machine to output the rest of the result!

O_5 (7G2)

S 2s 2s 2s 3s S S
2s 2s 2s 3s S S

S 2s S 2s S 2s 3s
2s S 2s S 2s 3s S
S 2s S 2s 3s S 2s
2s S 2s 3s S 2s S
S 2s 3s S 2s S 2s
2s 3s S 2s S 2s S
3s S 2s S 2s S 2s

—

O_6 (7G2) S S S 2s 2s 3s 2s
S S 2s 2s 3s 2s S
S 2s 2s 3s 2s S S
2s 2s 3s 2s S S S
2s 3s 2s S S S 2s
3s 2s S S S 2s 2s
2s S S S 2s 2s 3s

—

O_7 (7G2) S S S 2s 3s 2s 2s
S S 2s 3s 2s 2s S
S 2s 3s 2s 2s S S
2s 3s 2s 2s S S S
3s 2s 2s S S S 2s
2s 2s S S S 2s 3s
2s S S S 2s 3s 2s

—

O_8 (7G2) S S S 3s 2s 2s 2s
S S 3s 2s 2s 2s S
S 3s 2s 2s 2s S S
3s 2s 2s 2s S S S
2s 2s 2s S S S 3s
2s 2s S S S 3s 2s
2s S S S 3s 2s 2s

$O_9(7G_2)$ S 2s S S 2s 3s 2s
2s S S 2s 3s 2s S
S S 2s 3s 2s S 2s
S 2s 3s 2s S 2s S
2s 3s 2s S 2s S S
3s 2s S 2s S S 2s
2s S 2s S S 2s 3s

$O_{10}(7G_2)$ S 2s SS 3s 2s 2s
2s S S 3s 2s 2s S
S S 3s 2s 2s S 2s
S 3s 2s 2s S 2s S
3s 2s 2s S 2s S S
2s 2s S 2s S S 3s
2s S 2s S S 3s 2s

$O_{11}(7G_2)$
S 3s SS 2s 2s 2s
3s S S 2s 2s 2s S
S S 2s 2s 2s S 3s
S 2s 2s 2s S 3s S
2s 2s 2s S 3s S S
2s 2s S 3s S S 2s
2s S 3s S S 2s 2s

$O_{12}(7G_2)$
S 2s 2s S S 3s 2s
2s 2s S S 3s 2s S
2s S S 3s 2s S 2s
S S 3s 2s S 2s 2s
S 3s 2s S 2s 2s S
3s 2s S 2s 2s S S
2s S 2s 2s S S 3s

O₁₃ (7G₂)
S 2s 3s SS 2s 2s
2s 3s S S 2s 2s S
3s S S 2s 2s S 2s
S S 2s 2s S 2s 3s
S 2s 2s S 2s 3s S
2s 2s S 2s 3s S S
2s S 2s 3s S S 2s

O₁₄ (7G₂)
S 3s 2s SS 2s 2s
3s 2s S S 2s 2s S
2s S S 2s 2s S 3s
S S 2s 2s S 3s 2s
S 2s 2s S 3s 2s S
2s 2s S 3s 2s S S
~~2s 2s S 3s 2s~~
2s S 3s 2s S S 2s

O₁₅ (7G₂)
S 2s 2s 3s S S 2s
2s 2s 3s S S 2s S
2s 2s S S 2s S 2s
3s S S 2s S 2s 2s
S S 2s S 2s 2s 3s
S 2s S 2s 2s 3s S
2s S 2s 2s 3s S S

O₁₆ (7G₂)
S 2s 3s 2s SS 2s
2s 3s 2s S S 2s S
3s 2s S S 2s S 2s
2s S S 2s S 2s 3s
S S 2s S 2s 3s 2s
S 2s S 2s 3s 2s S
2s S 2s 3s 2s S S

O_{17} (7G_2) S 3s 2s 2s S S 2s
3s 2s 2s S S 2s S
2s 2s S S 2s S 3s
2s S S 2s S 3s 2s
S S 2s S 3s 2s 2s
S 2s S 3s 2s 2s S
2s S 3s 2s 2s S S
———

O_{18} (7G_2) S 2s S 2s S 3s 2s
2s S 2s S 3s 2s S
S 2s S 3s 2s S 2s
2s S 3s 2s S 2s S
S 3s 2s S 2s S 2s
3s 2s S 2s S 2s S
2s S 2s S 2s S 3s
—

O_{19} (7G_2) S 2s S 3s S 2s 2s
2s S 3s S 2s 2s S
S 3s S 2s 2s S 2s
3s S 2s 2s S 2s S
S 2s 2s S 2s S 3s
2s 2s S 2s S 3s S
2s S 2s S 3s S 2s
—

O_{20} (7G_2) S 3s S 2s S 2s 2s
3s S 2s S 2s 2s S
S 2s S 2s 2s S 3s
2s S 2s 2s S 3s S
S 2s 2s S 3s S 2s
2s 2s S 3s S 2s S
2s S 3s S 2s S 2s

Food for Thought...

Now looking at such a huge number of possibilities in scales, one may wonder as to why can't the pure treatment of Rāgas still not be encompassed within such a broad scope? Before deciding upon that let's go step by step in graduating from the outlook of 5, 6 or 7 uniformly in the ascent & decent of the scale, to further broadening it to the ascent and descent in 2 different quantities of orders i.e. Audav-Sampoorna or 5 in the ascent & 7 in the descent. For example consider the Rāga Shuddha-Kalyān as S R G P D **S** in its ascent and in its descent **S** N D P M G R S, or in terms of the gaps:

$$\text{(0)} \quad +2s+2s+3s+2s+3s \quad \text{in other} \quad R_x [O_y [5 G_i]$$
$$\text{(12s)} \quad -s-2s-2s-s-2s-2s-2s \quad \text{words,} \quad R_x [O_y [7 G_i]$$

with at least 5 x 7 = 35 Rotations with these 2 sequences or Orders rotated. Then if we shuffle the sequences along with their rotations, it gives 5C2 x 7C2 = 10 x 21 = 210 Rotations or scales! Now this 1st step is only valid assuming your scales have notes ascending & descending in a continuous succession of notes. But what if that is not true in other cases of "Audav-Sampoorna" i.e, Bāgeshree, Shyām-Kalyān etc where either the ascent or the descent isn't typically in a continuous succession of notes i.e. 'vakra' or 'zigzag'. For example in Rāga Bāgeshree **S** N D, m P D, m g R S or in Shyām-Kalyān **S** N D P, M P, G M R S. Or for 'zigzag' patterns in both ascent & descent for example, Shudha-Vasant: S m, G M G, M D N **S** in the ascent and in the descent **S** N D, M m G, M G r S. So how are going to put examples like these in a counting mechanism in order to list out the possibilities. Using Shuddha-Vasant, when you try do the rotation should it begin with r or with m as 2nd **R_x**?! So that's where I feel it that it's not required to overdo the scale system. But if anyone wants to still purse it for whatever purpose, be my guest!

For me it has always been **a way to derive scales that I may have never imagined with my background in Sitar** and yet I may be able to play them on my SiGui as a chromatic instrument or any other chromatic fretted Sitar, not the usual gap in frets between S & R, P & D, etc. **This has to be the way forward instead of accepting limitation in an immature understanding of the traditional Sitar, in trying to convince the world using a very interesting psychology(!)** that by acknowledging at first the limitations of those missing frets, "Those scales are difficult to play on the Sitar as it's not chromatic"(!) and then for some weird reason attempt only a few of the scales by adjusting the missing frets by pulling or bending on another fret, and even more ridiculous to brag "See how we are adjusting by pulling the main string where there are no frets!". Then why don't they attempt let alone brag about it on the second string? I'll tell you. Because the copper string (2nd) will snap instantly by pulling. Or for those who play the bass string pointlessly on a few frets that are in tune as **the cultural world tries very hard to dismiss my SiGui / SiVee, which has solved this 150 years old problem of volatility with the fret tuning for the bass strings.** Obviously all those who have rejected the concept of playing bass strings even today, in 2023, on the Sitar incorporated from the 7-string system of the Veena from 1870s, can remain stuck as their "ignorance is bliss"! The so-called 'adjustments' in a few of the scales versus an ocean that one can fathom using my notion of a scale as defined by **$R_x [O_y [n G_i]$,**
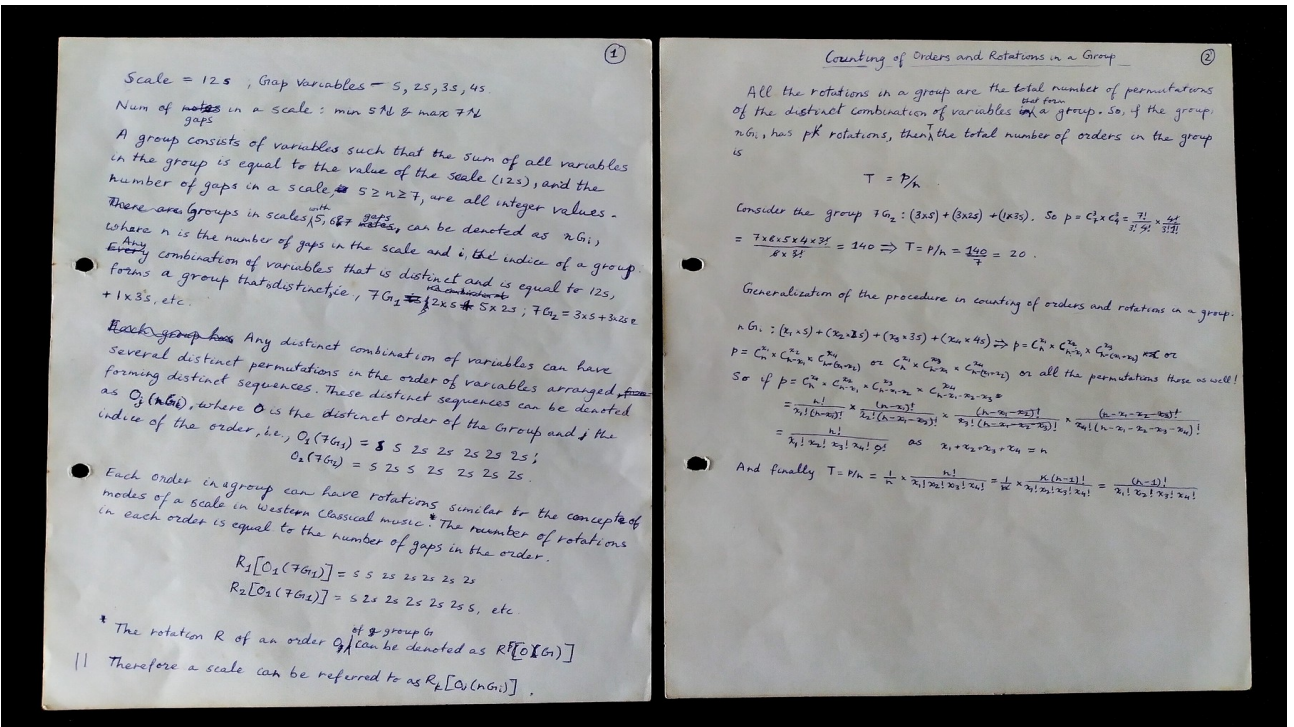
thanks to blessing of the COWS!!!! 🙏🐮🐄🐃🙏
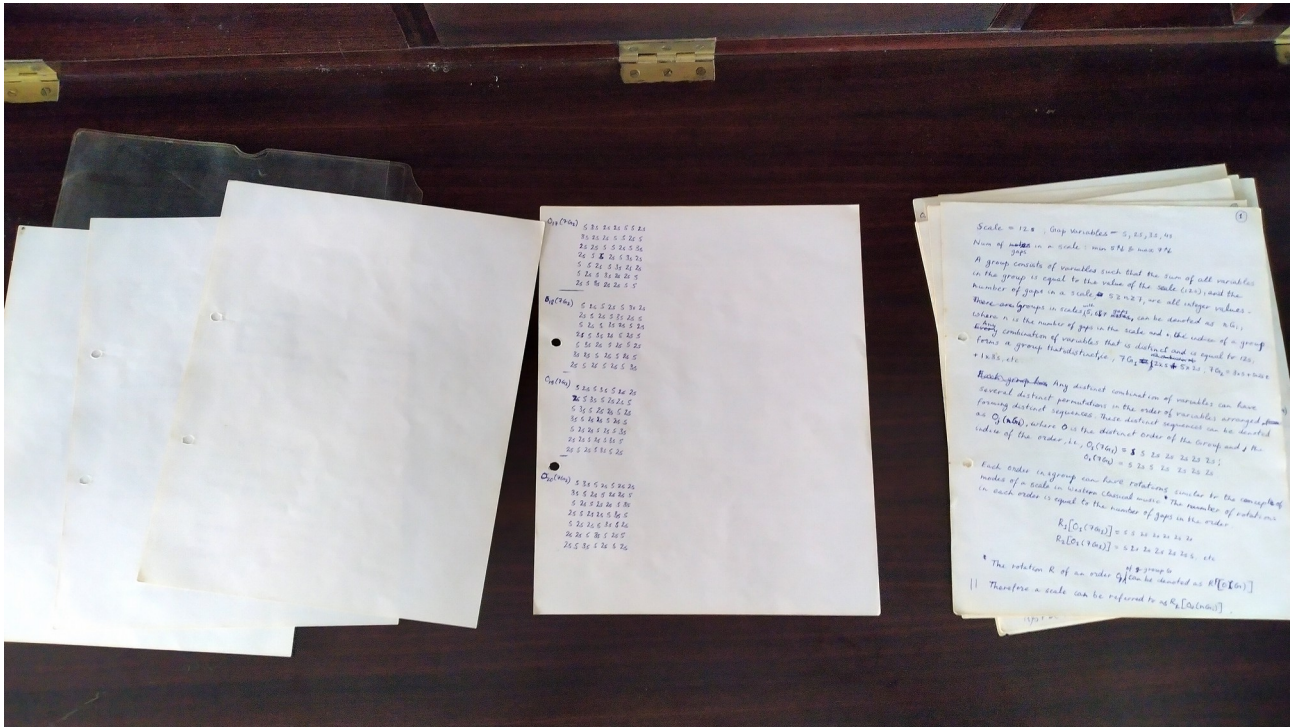
Saved 3 Rough Papers(!) as I thought I will get over my laziness to code...

**... one day maybe! Until then remember that this is
my mathematical tool to generate your scales!**

Page ①

Scale = 12 s , Gap Variables — s, 2s, 3s, 4s.

Num of ~~notes~~ gaps in a scale : min 5 & max 7.

A group consists of variables such that the sum of all variables in the group is equal to the value of the scale (12s), and the number of gaps in a scale, $5 \geq n \geq 7$, are all integer values.

~~There are~~ Any groups in scales (5, 6, & 7 gaps) can be denoted as nG_i, where n is the number of gaps in the scale and i the indice of a group. ~~Every~~ Any combination of variables that is distinct and is equal to 12s, forms a group that is distinct, ie, $7G_1 = 2 \times s + 5 \times 2s$, $7G_2 = 3 \times s + 3 \times 2s + 1 \times 3s$, etc.

~~Each group has~~ Any distinct combination of variables can have several distinct permutations in the order of variables arranged, forming distinct sequences. These distinct sequences can be denoted as $O_j(nG_i)$, where O is the distinct order of the Group and j the indice of the order, ie, $O_1(7G_1) = s \ s \ 2s \ 2s \ 2s \ 2s \ 2s$, $O_2(7G_1) = s \ 2s \ s \ 2s \ 2s \ 2s \ 2s$.

Each order in a group can have rotations similar to the concept of modes of a scale in western Classical music.* The number of rotations in each order is equal to the number of gaps in the order.

$$R_1[O_1(7G_1)] = s \ s \ 2s \ 2s \ 2s \ 2s \ 2s$$
$$R_2[O_1(7G_1)] = s \ 2s \ 2s \ 2s \ 2s \ 2s \ s, \text{ etc.}$$

* The rotation R of an order Oj of a group Gi can be denoted as $R_k[O_j(nG_i)]$

‖ Therefore a scale can be referred to as $R_k[O_j(nG_i)]$.

Page ②

Counting of Orders and Rotations in a Group

All the rotations in a group are the total number of permutations of the distinct combination of variables that form a group. So, if the group, nG_i, has p rotations, then the total number of orders in the group is

$$T = P/n$$

Consider the group $7G_2$: $(3 \times s) + (3 \times 2s) + (1 \times 3s)$. So $p = C_7^3 \times C_4^3 = \frac{7!}{3! \ 4!} \times \frac{4!}{3! \ 1!}$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{6 \times 3!} = 140 \implies T = P/n = \frac{140}{7} = 20.$$

Generalization of the procedure in counting of orders and rotations in a group.

nG_i : $(z_1 \times s) + (z_2 \times 2s) + (z_3 \times 3s) + (z_4 \times 4s) \implies p = C_n^{z_1} \times C_{n-z_1}^{z_2} \times C_{(n-z_1-z_2)}^{z_3}$ etc

$p = C_n^{z_1} \times C_{n-z_1}^{z_2} \times C_{(n-z_1-z_2)}^{z_3}$ or $C_n^{z_4} \times C_{n-z_4}^{z_3} \times C_{(n-z_4-z_3)}^{z_2}$ or all the permutations those as well!

So if $p = C_n^{z_1} \times C_{n-z_1}^{z_2} \times C_{n-z_1-z_2}^{z_3} \times C_{n-z_1-z_2-z_3}^{z_4}$

$$= \frac{n!}{z_1! (n-z_1)!} \times \frac{(n-z_1)!}{z_2! (n-z_1-z_2)!} \times \frac{(n-z_1-z_2)!}{z_3! (n-z_1-z_2-z_3)!} \times \frac{(n-z_1-z_2-z_3)!}{z_4! (n-z_1-z_2-z_3-z_4)!}$$

$$= \frac{n!}{z_1! \ z_2! \ z_3! \ z_4! \ 0!} \text{ as } z_1 + z_2 + z_3 + z_4 = n$$

And finally $T = P/n = \frac{1}{n} \times \frac{n!}{z_1! z_2! z_3! z_4!} = \frac{1}{k} \times \frac{k(n-1)!}{z_1! z_2! z_3! z_4!} = \frac{(n-1)!}{z_1! z_2! z_3! z_4!}$

So what had happened in my Math Life?

*To the best of my knowledge, I was kicked out of class, for my negligence (often in simple arithmetic or my difficulty with understanding French as the medium in which it was taught as I simply "flew in my self-study" later using Math books in English even as I respected that it was taught in French as I had heard that The Mother wanted it that way) **all of which my teacher interpreted it as** my lack of level in Math(?), "**votre niveau de mathématique n'est pas suffisant**" further ensuring that I would never be allowed to take up mainstream Math! Even as another **classmate and I were the only 2 students in the class who went up to the board to solve problems** on the spot, while our **dear teacher** would invariably **check** in his college days **notebook for solutions** to those problems**, in a manner most 'discreet' in trying his best to hide his notebook under his desk but forgetting that we could notice his eyes regularly gaze down to check his notebook and raising them up to quickly see if any of us caught him in the act! Neither could he learn those answers by heart and nor could he derive the solutions on the spot, **even after 10 years of his teaching Math** and yet he chose to judge our level in Math! **I don't mean to dig into the past to seek any justice**, as I had to play by his rules of passing some intermediary tests other than submitting homework and our aptitude in class, with his supposedly weighing all 3 aspects equally(!) as per his own claims initially with his extremely 'fair' system of allotting 2 points for a problem solved with 1.5 for the answer irrespective of the method and only 0.5 for the right method! As I would very often get the method right but neglect in crosschecking my arithmetic in the final answer I was 'lucky' to get only 0.5 points! **The intermediary tests were not necessarily a common practice with the other teachers,** more able than him, who may not rely on their old notebooks unlike him(!) while teaching Math at the same level or higher, invariably leading to the question of what should be the common practice in teaching and whether that is aligned with the spirit of learning in the said environment.*

*There was this one time when **I had to go to the board to solve** a complex number algebra problem. **I used my original approach** which was a little longer in the number of steps, **as opposed to the shorter method that he only knew thanks to his notebook!!!** And he could not even hide his notebook given the effort in solving the problem seriously exposing his own level in Math or the lack of it! As none of the other students had got either of the methods (mine or his) **HE WAS FORCED TO TELL ME, "Bonne idée!"** ONLY TO CONTRADICT HIMSELF after a month in claiming that my math level was in question??*

*It's probably 'illogical' to use power to label anyone unfit in Math barely in the first 2 months of the year, **and one can really wonder whether by kicking me out of class, did he really hamper my learning? Just as much as knowing that my CONTINUING with his class MIGHT'VE ACTUALLY HAMPERED my learning in Math as I experienced progress independently!** Even if he enjoyed the position of approving certificates for "deserving students"! Was he possibly wasting his time in teaching just 1 more student in a class of 6 others as opposed to some teacher outside Ashram, all alone teaching some hundred students perhaps out of a "less competitive purpose"?! Learning Math never stopped for me and **most importantly I learnt not to care for a piece of paper as in a certificate beyond these rough papers on which I discovered my solution independently!** I had studied further on my own and I seriously doubt whether he might have been even capable of teaching me at that level. **However, as a student of SAICE I will remain grateful to him for having learnt from him at least the topic concerned in this research(!) irrespective of how brilliantly I've applied it here(!); irrespective of my being capable of learning it independently** or from elsewhere **and irrespective of the right & wrong of things i.e.** politics, higher pressure etc, if any.*